Solution of nonlinear coupled problems of flow induced vibrations of an elastic structure

Řešení nelineárních problémů interakce těles a tekutin

Sváček, P., M. Feistauer, J. Horáček

Czech Technical University,
Faculty of Mechanical Engineering,
Department of Technical Mathematics

PANM 2010

Cíle přednášky

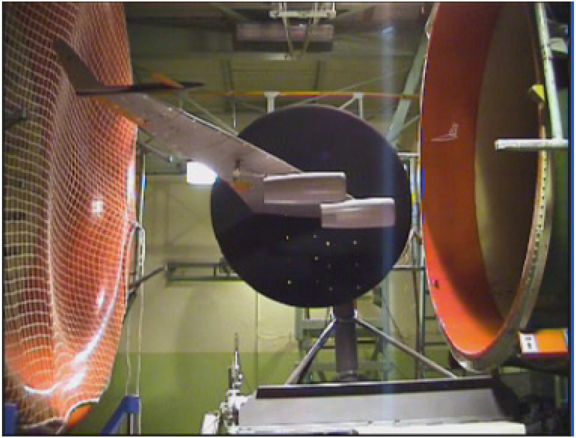
- Charakterizace a popis problémů FSI
- Popis matematického modelu, jednoduché odhady
- Popis numerického modelu
- Ukázky řešených problémů

Sdružené problémy v praktických aplikacích

- aeroelastické problémy v letectví (bezpečnost letadel,...)
- aeroelastické/hydroelastické problémy ve stavebnictví (stabilita mostů, vysokých budov, apod.)
- biomechanické problémy

Sdružené problémy v praktických aplikacích

Při návrhu letadel



Měření VZLU Letnany

Certifikace letadel (analyticky i experimentálně)

V současnosti: Aktivní kontrola nestabilit, bezpilotní letadla (vliv poryvů větru), atp.

Sdružené problémy v praktických aplikacích

Tacoma Narrows Bridge Nov. 7, 1940

Zdroj: internet.
Hlavní příčina: rezonanční frekvence

Sdružené problémy v praktických aplikacích

Chladící věže Ferrybridge, Yorkshire, Nov 1st, 1965

- 3 chladící věže zničeny vibracemi způsobenými větrem
- nevhodné seskupení věží

Most, Volgograd 2010

most 7 km pres reku Volhu, 2010

Zdroj: internet, www.novinky.cz

Aeroelastický model: pohyb struktury, proudění, podmínky na rozhraní Jaké jsou cíle modelování:

- detekce aeroelastických nestabilit (flutter boundary)
- chování po ztrátě stability
- vliv nelinearit (např. v biomechanice)

řeší se zjednodušený problém např. **detekce nestability**

Detekce nestability

Závisí na typu

Členění dle způsobu vzniku *Naudasher-Rockwell, 1994*

1. **Externaly induced excitation**

   vibrace vybuzene z vnějšku

2. **Movement-induced excitation**

   nestabilita v důsledku vysoké rychlosti proudění (**flutter**)

3. **Instability-induced excitation**

   v důsledku rezonance

např. detekce tzv. flutter boundary

Aeroelastic problems

Obyklé metody pro rešení: linearizace aerodynamických sil

- **Analytical methods** (Theodorsen theory)

$$I_\alpha \ddot{\alpha} + d_\alpha \dot{\alpha} + k_\alpha \alpha = M_3$$

$$I_\alpha \ddot{\alpha} + d_\alpha \dot{\alpha} + k_\alpha \alpha = M_3 \approx U_\infty^2 (c_1 \ddot{\alpha} + c_2 \dot{\alpha} + c_3 k_\alpha \alpha)$$

$$(I_\alpha - U_\infty^2 c_1)\ddot{\alpha} + (d_\alpha - U_\infty^2 c_2)\dot{\alpha} + (k_\alpha - U_\infty^2 c_3)\alpha = 0$$

- řešení závisí na parametru U_∞
- Tvar řešení $\alpha = e^{\omega t}$, $\omega = D + i\,F$
- Jiný přístup: **Computational aeroelasticity**

Charakterizace problému

- nestacionární 3D proudění na pohyblivé oblasti
- 3D pohyb elastického tělesa
- post-flutter: velké výchylky a nelinearity
- podmínky na rozhraní prostředí

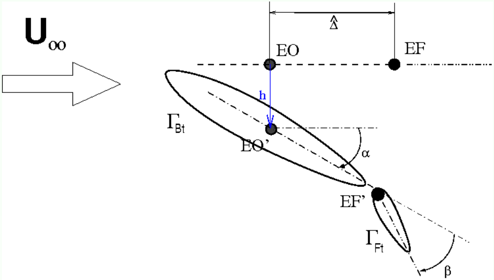
Zjednodušený model

- 2D proudění
- těleso popsané pomocí několika stupňů volnosti **(3dof) or (2dof)**

Structure model (3dof)

14/ 65

Equations of motion

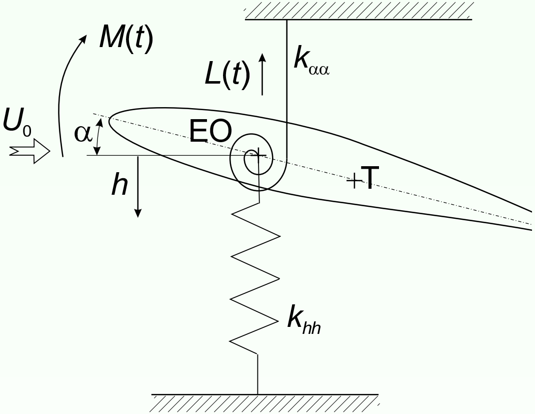


$$\begin{pmatrix} m & S_\alpha & S_\beta \\ S_\alpha & I_\alpha & S_1 \\ S_\beta & S_1 & I_\beta \end{pmatrix} \begin{pmatrix} \ddot{h} \\ \ddot{\alpha} \\ \ddot{\beta} \end{pmatrix} + \begin{pmatrix} k_h & 0 & 0 \\ 0 & k_\alpha & 0 \\ 0 & 0 & k_\beta \end{pmatrix} \begin{pmatrix} h \\ \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -L \\ M_3 \\ M_\beta \end{pmatrix}$$

where $S_1 = \widetilde{\Delta} S_\beta + I_\alpha$, $m, S_\alpha, I_\alpha, S_\beta, I_\beta$ - structural parameters.

geometrical nonlinearities? `Horacek et al, App Cmp Mech, 2007`

Structure model(2dof) - nonlinear



Airfoil motion equations

system ODEs

$$\begin{aligned} m\ddot{h} + S_\alpha \ddot{\alpha} \cos\alpha - S_\alpha \dot{\alpha}^2 \sin\alpha + K_{hh} h &= -L \\ S_\alpha \ddot{h} \cos\alpha + I_\alpha \ddot{\alpha} + K_{\alpha\alpha}\alpha &= M_3 \end{aligned}$$

Fluid model

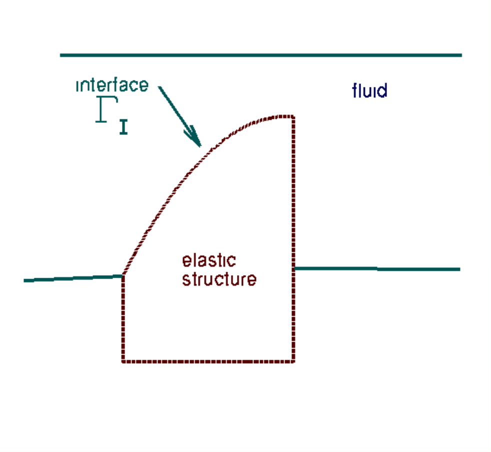
- incompressible viscous fluid
- Reynolds numbers $Re = 10^3 - 10^7$
- non-stationary 2D discretization

Navier-Stokes system

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla p - \nu \triangle \mathbf{v} &= \mathbf{f} \\ \nabla \cdot \mathbf{v} &= 0 \quad \text{in } \Omega_t \end{aligned}$$

$\mathbf{v}$ - fluid velocity, p - kinematic pressure, ν - kinematic viscosity
Solution on Ω_t

Interface conditions

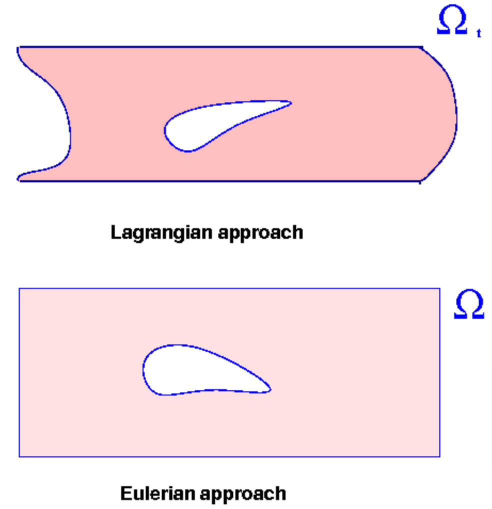


- kinematic conditions $\mathbf{v} = \dot{\mathbf{u}}$
- dynamic conditions
- How to treat moving domain?
  Arbitrary Lagrangian-Eulerian(ALE) method!

Moving meshes

How to approximate flow on moving mesh?

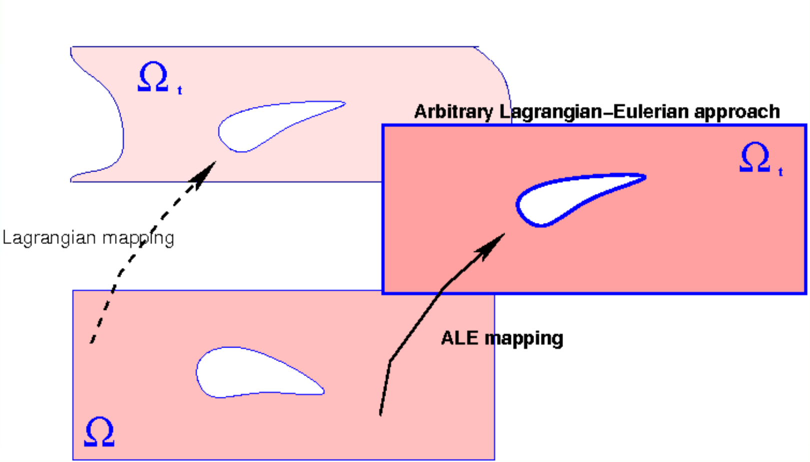
Eulerian $\times$ Lagrangian approaches



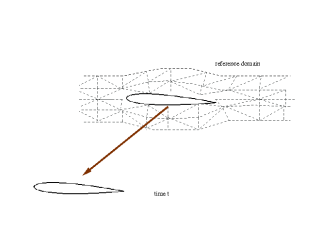
Moving meshes

How to approximate flow on moving mesh?

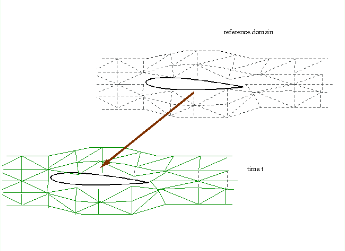
Eulerian $\times$ Lagrangian approaches



How to apply ALE method?

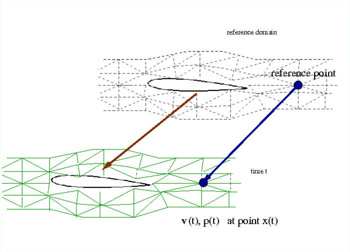


How to apply ALE method?



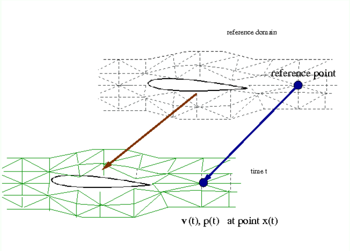
- ALE mapping $\mathcal{A}_t : \Omega_0 \mapsto \Omega_t$

How to apply ALE method?



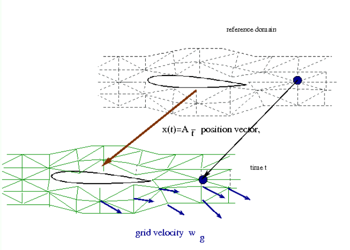
- ALE mapping $\mathcal{A}_t : \Omega_0 \mapsto \Omega_t$
- $\frac{D^{\mathcal{A}} f}{Dt}$ - ALE derivative,

How to apply ALE method?



- ALE mapping $\mathcal{A}_t : \Omega_0 \mapsto \Omega_t$
- $\frac{D^{\mathcal{A}} f}{Dt}$ - ALE derivative, point $x(t)$ moves in time

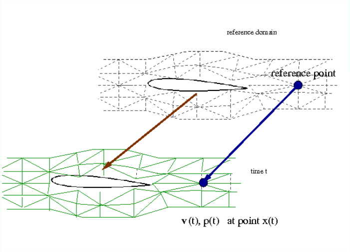
How to apply ALE method?



- ALE mapping $\mathcal{A}_t : \Omega_0 \mapsto \Omega_t$
- $\frac{D^{\mathcal{A}} f}{Dt}$ - ALE derivative, point $x(t)$ moves in time
- domain velocity

$$\mathbf{w}_D(x, t) = \tilde{\mathbf{w}}_D(\xi, t) = \frac{\partial \mathcal{A}_t(\xi)}{\partial t}$$

How to apply ALE method?



- ALE mapping $\mathcal{A}_t : \Omega_0 \mapsto \Omega_t$
- $\frac{D^{\mathcal{A}} f}{Dt}$ - ALE derivative, point $x(t)$ moves in time
- domain velocity

$$\mathbf{w}_D(x,t) = \tilde{\mathbf{w}}_D(\xi,t) = \frac{\partial \mathcal{A}_t(\xi)}{\partial t}$$

- relation

$$\frac{D^{\mathcal{A}} f}{Dt} = \frac{\partial f}{\partial t} + (\mathbf{w}_D \cdot \nabla) f$$

Using

$$\frac{D^{\mathcal{A}}}{Dt} f = \frac{\partial f}{\partial t} + (\mathbf{w}_D \cdot \nabla) f, \tag{1}$$

ALE non-conservative form

$$\begin{aligned} \frac{D^{\mathcal{A}}\mathbf{v}}{Dt} + (\mathbf{v} - \mathbf{w}_D) \cdot \nabla \mathbf{v} + \nabla p - \nu \triangle \mathbf{v} &= \mathbf{f} \\ \nabla \cdot \mathbf{v} &= 0 \quad \text{in } \Omega_t \end{aligned}$$

Mathematical model

- ALE form of Navier-Stokes equations

$$\frac{D^{\mathcal{A}}\mathbf{v}}{Dt} + ((\mathbf{v} - \mathbf{w}_D) \cdot \nabla)\,\mathbf{v} - \nabla \cdot \left(\nu(\nabla\mathbf{v} + \nabla^T\mathbf{v})\right) + \nabla p = 0$$

- structure model $\vec{u}$ - generalized coordinates

$$\mathbb{M}\ddot{\vec{u}} + \mathbb{D}\dot{\vec{u}} + \mathbb{K}\vec{u} = \mathbf{f}$$

- interface conditions
- ALE method - deformation of the domain depends on $\vec{u}$

Mathematical formulation of FSI problem

- Find $\mathbf{v} = \mathbf{v}(x, t)$ and $p = p(x, t)$

$$\frac{D^{\mathcal{A}}\mathbf{v}}{Dt} + ((\mathbf{v} - \mathbf{w}_D) \cdot \nabla)\,\mathbf{v} - \nu \triangle \mathbf{v} + \nabla p = 0$$

$$\nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega_t,$$

- with b.c. $\mathbf{v} = \dot{\mathbf{u}}$, and find $\mathbf{u} = \mathbf{u}(\xi, t)$

$$\mu \triangle \mathbf{u} + (\mu + \lambda)\nabla(\nabla \cdot \mathbf{u}) = 0 \text{ in } \Omega_0$$

- with b.c. $\mathbf{u} = \mathbf{u}(\alpha, h)$, and find $\alpha = \alpha(t)$, $h = h(t)$

$$m\ddot{h} + S_\alpha \ddot{\alpha} + d_{hh}\dot{h} + K_h h = -L = \int_{\Gamma_{Wt}} \tau_{2j} n_j dS,$$

$$S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + d_{\alpha\alpha}\dot{\alpha} + K_\alpha \alpha = M_3 = \int_{\Gamma_{Wt}} r_i^{ort} \tau_{ij} n_j dS,$$

- **For application of FEM $\rightarrow$ Weak formulation!**

Weak formulation

- Find $\mathbf{v}(\cdot, t) \in \mathbf{v}_D + \mathbf{w}_D + \mathbf{H}_0^1(\Omega_t)$ and $p(\cdot, t) \in L^2(\Omega_t)$

$$\left(\frac{D^{\mathcal{A}}\mathbf{v}}{Dt}, \mathbf{z}\right)_{\Omega_t} + \Big([(\mathbf{v} - \mathbf{w}_D) \cdot \nabla]\, \mathbf{v}, \mathbf{z}\Big)_{\Omega_t} + (\nu \nabla \mathbf{v}, \nabla \mathbf{z})_{\Omega_t}$$
$$-(p, \nabla^T \cdot \mathbf{z})_{\Omega_t} + (\nabla \cdot \mathbf{v}, q)_{\Omega_t} = 0,$$

- and $\mathbf{u} \in \mathbf{u}_{\Gamma_{W_t}} + \mathbf{H}_0^1(\Omega_0)$,

$$\mu\Big(\nabla \mathbf{u}, \nabla \mathbf{N}\Big)_{\Omega_0} + (\mu + \lambda)\Big((\nabla \cdot \mathbf{u}), \nabla \cdot \mathbf{N}\Big)_{\Omega_0} = 0$$

- ODEs for α, h with

$$L = -\int_{\Gamma_{W_t}} \tau_{2j} n_j dS,$$

$$M_3 = \int_{\Gamma_{W_t}} r_i^{ort} \tau_{ij} n_j dS,$$

- L and M_3 - requires weak formulation

Weak formulation - fluid forces

- fluid force acting on the profile

$$F_i = -\int_{\Gamma_W} \sum_{j=1}^{2} \tau_{ij} n_j dS, \qquad i = 1, 2.$$

Weak formulation - fluid forces

- fluid force acting on the profile

$$F_i = -\int_{\Gamma_W} \sum_{j=1}^{2} \tau_{ij} n_j dS, \qquad i = 1, 2.$$

- stress tensor

$$\tau_{ij} = \rho \left(-p\delta_{ij} + \nu \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) \right)$$

Weak formulation - fluid forces

- fluid force acting on the profile

$$F_i = -\int_{\Gamma_W} \sum_{j=1}^{2} \tau_{ij} n_j dS, \qquad i = 1, 2.$$

- stress tensor

$$\tau_{ij} = \rho \left(-p\delta_{ij} + \nu \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) \right)$$

- start with momentum of equations

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^{2} \frac{\partial(\rho v_i v_j)}{\partial x_j} = \sum_{j=1}^{2} \frac{\partial(\tau_{ij})}{\partial x_j}, \qquad i = 1, 2.$$

Weak formulation - fluid forces

- fluid force acting on the profile

$$F_i = -\int_{\Gamma_W} \sum_{j=1}^{2} \tau_{ij} n_j dS, \qquad i = 1, 2.$$

- stress tensor

$$\tau_{ij} = \rho \left(-p\delta_{ij} + \nu \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) \right)$$

- start with momentum of equations

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^{2} \frac{\partial(\rho v_i v_j)}{\partial x_j} = \sum_{j=1}^{2} \frac{\partial(\tau_{ij})}{\partial x_j}, \qquad i = 1, 2.$$

- use the test function φ, integrate and use Green's theorem

Weak formulation - fluid forces

- fluid force acting on the profile

$$F_i = -\int_{\Gamma_W} \sum_{j=1}^{2} \tau_{ij} n_j dS, \qquad i = 1, 2.$$

- stress tensor

$$\tau_{ij} = \rho \left(-p\delta_{ij} + \nu \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) \right)$$

- start with momentum of equations

$$\frac{\partial(\rho v_i)}{\partial t} + \sum_{j=1}^{2} \frac{\partial(\rho v_i v_j)}{\partial x_j} = \sum_{j=1}^{2} \frac{\partial(\tau_{ij})}{\partial x_j}, \qquad i = 1, 2.$$

- use the test function φ, integrate and use Green's theorem
- we get

$$-F_i = \int_\Omega \frac{\partial(\rho v_i)}{\partial t} \varphi \, dx + \int_\Omega \sum_{j=1}^{2} \frac{\partial(\rho v_i v_j)}{\partial x_j} \varphi \, dx + \int_\Omega \sum_{j=1}^{2} \tau_{ij} \frac{\partial \varphi}{\partial x_j} \, dx,$$

Weak formulation

- Find $\mathbf{v}(\cdot, t) \in \mathbf{v}_D + \mathbf{w}_D + \mathbf{H}_0^1(\Omega_t)$ and $p(\cdot, t) \in L^2(\Omega_t)$

$$\left(\frac{D^{\mathcal{A}}\mathbf{v}}{Dt}, \mathbf{z}\right)_{\Omega_t} + \left(\left[(\mathbf{v} - \mathbf{w}_D) \cdot \nabla\right] \mathbf{v}, \mathbf{z}\right)_{\Omega_t} + (\nu \nabla \mathbf{v}, \nabla \mathbf{z})_{\Omega_t}$$
$$-(p, \nabla^T \cdot \mathbf{z})_{\Omega_t} + (\nabla \cdot \mathbf{v}, q)_{\Omega_t} = 0,$$

- and $\mathbf{u} \in \mathbf{u}_{\Gamma_{W_t}} + \mathbf{H}_0^1(\Omega_0)$,

$$\mu\left(\nabla \mathbf{u}, \nabla \mathbf{N}\right)_{\Omega_0} + (\mu + \lambda)\left((\nabla \cdot \mathbf{u}), \nabla \cdot \mathbf{N}\right)_{\Omega_0} = 0$$

- ODEs for α, h with

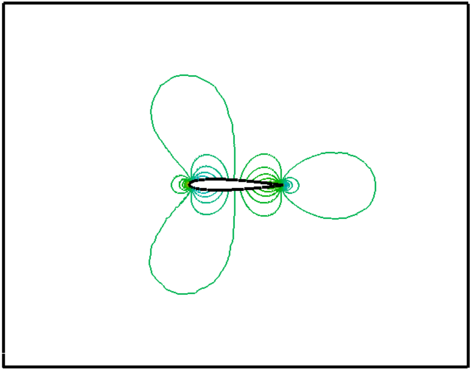
$$-L = \int_{\Omega_t} \frac{\partial(\rho v_2)}{\partial t} \varphi \, dx + \int_{\Omega_t} \frac{\partial(\rho v_2 v_j)}{\partial x_j} \varphi \, dx + \int_{\Omega_t} \tau_{2j} \frac{\partial \varphi}{\partial x_j} \, dx,$$

$$-M_3 = \int_{\Omega_t} \frac{\partial(\rho v_i)}{\partial t} r_i^{ort} \varphi \, dx + \int_{\Omega_t} \frac{\partial(\rho v_i v_j)}{\partial x_j} \varphi r_i^{ort} \, dx + \int_{\Omega_t} \tau_{ij} \frac{\partial r_i^{ort} \varphi}{\partial x_j} \, dx.$$

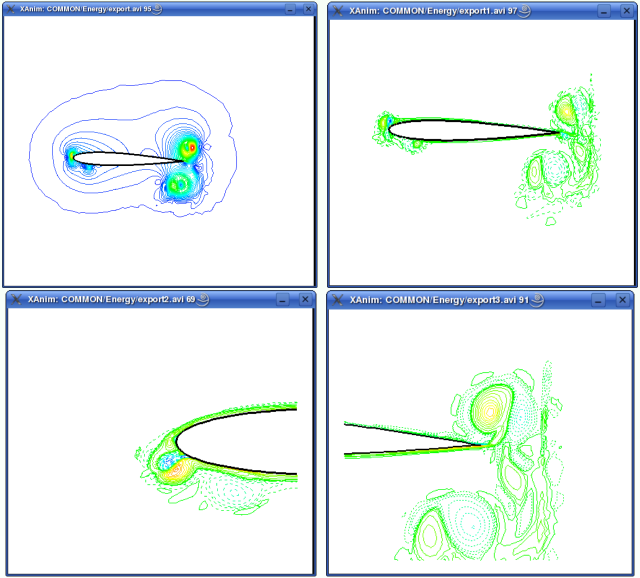
- Energy estimate: Choose $\varphi = \mathbf{v}$, but $\mathbf{v} \neq 0$ on Dirichlet part of the boundary.

Test Problem

- no inlet, outlet ($\mathbf{v} = 0$ on $\partial\Omega \setminus \Gamma_{Wt}$)
- $\mathbf{v} = \dot{\mathbf{u}} = \mathbf{w}_D$ on Γ_{Wt} - flexibly supported airfoil
- Initial condition: $\alpha(0) = \alpha_0, h(0) = h_0$, others $\rightarrow$ zeros.



Test Problem - Energy Estimate



Test Problem - Energy estimate

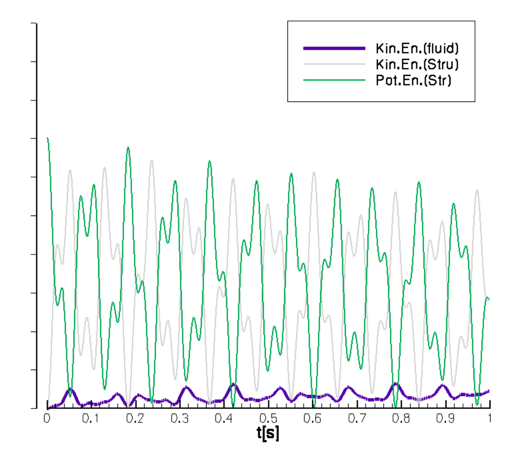
- Choose $\varphi = \mathbf{v}$, assume $\mathbf{v} = \mathbf{v}_D \equiv 0$ on $\partial\Omega_t \setminus \Gamma_{Wt}$, then

$$\frac{d}{dt}\int_{\Omega_t} \frac{1}{2}\rho|\mathbf{v}|^2 dx + \frac{d}{dt}(\frac{1}{2}k_h h^2 + \frac{1}{2}k_\alpha \alpha^2) + \frac{d}{dt}\frac{1}{2}(m\dot{h}^2 + 2S_\alpha \cos\alpha \dot{h}\dot{\alpha} + I_\alpha \dot{\alpha}^2) + \nu\rho\int_{\Omega_t} |\nabla\mathbf{v}|^2 dx = 0$$

Test Problem - Energy Estimate

Přenosy energie mezi tekutinou a tělesem jsou podstatné!

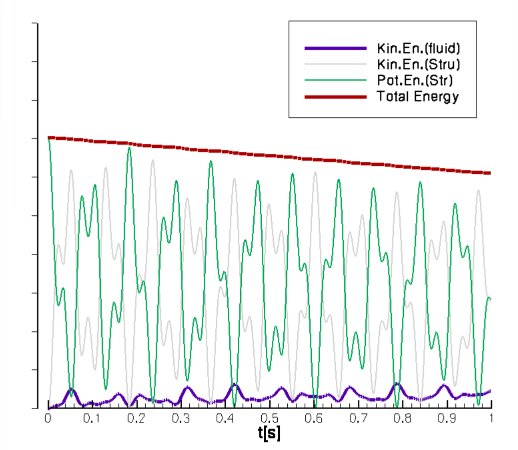
$$\frac{1}{2}\frac{d}{dt}\left[\left(\int_{\Omega_t}\rho|\mathbf{v}|^2 dx\right) + (k_h h^2 + k_\alpha \alpha^2) + (m\dot{h}^2 + 2S_\alpha \cos\alpha \dot{h}\dot{\alpha} + I_\alpha \dot{\alpha}^2)\right] \leq 0$$



Test Problem - Energy Estimate

Přenosy energie mezi tekutinou a tělesem jsou podstatné!

$$\frac{1}{2}\frac{d}{dt}\left[\left(\int_{\Omega_t}\rho|\mathbf{v}|^2dx\right)+(k_h h^2+k_\alpha\alpha^2)+(m\dot{h}^2+2S_\alpha\cos\alpha\,\dot{h}\dot{\alpha}+I_\alpha\dot{\alpha}^2)\right]\leq 0$$



Numerical Approximation

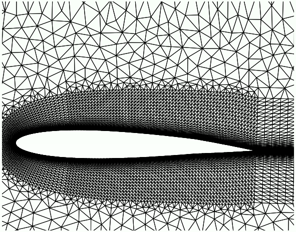
- ALE mapping construction
- time discretization
- space discretization

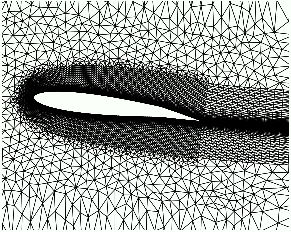
MESH MOVING STRATEGY

- **linear elasticity analogy**

$$-\sum_j \frac{\partial \sigma_{ij}(\mathbf{u})}{\partial x_j} = 0, \qquad \sigma_{ij} = \lambda \left(\sum_k \varepsilon_{kk} \right) \delta_{ij} + 2\mu\varepsilon_{ij},$$

- Lamé const. λ, μ depends on E, σ
- E modulus of elasticity, σ the Poisson ratio





Grid deformation **AVI format**

The ALE derivative approximation

Navier-Stokes system in ALE (nonconservative) form

$$\begin{aligned} \frac{D^{\mathcal{A}}}{Dt}\mathbf{v} + (\mathbf{v} - \mathbf{w}_D) \cdot \nabla \mathbf{v} + \nabla p - \nu \triangle \mathbf{v} &= 0 \\ \nabla \cdot \mathbf{v} &= 0 \quad \text{in } \Omega_t \end{aligned}$$

- approximate

$$\begin{aligned} \frac{D^{\mathcal{A}}\mathbf{v}}{Dt} &\approx \frac{3\mathbf{v}_{n+1} - 4\tilde{\mathbf{v}}_n + \tilde{\mathbf{v}}_{n-1}}{2\Delta t} \\ \mathbf{w}_D(X(t_{n+1}), t_{n+1}) &\approx \frac{3X_{n+1} - 4X_n + X_{n-1}}{2\Delta t} \end{aligned}$$

  depends on ALE method, FV or FEs, GCL

- $\tilde{\mathbf{v}}_n$ and $\tilde{\mathbf{v}}_{n-1}$ transformed on Ω_{n+1}

Geometrical Conservation Law

For FVs:

- Use equation $W_t + \nabla \cdot F(W) = 0$

$$\frac{d}{dt}\int_{D(t)} W dx + \int_{\partial D(t)} \left(\vec{F}(W) - \vec{\mathbf{w}}_D W\right) \cdot \vec{n} dS = 0$$

Geometrical Conservation Law

For FVs:

- Use equation $W_t + \nabla \cdot F(W) = 0$

$$\frac{d}{dt}\int_{D(t)} W dx + \int_{\partial D(t)} \left(\vec{F}(W) - \vec{\mathbf{w}_D} W\right) \cdot \vec{n} dS = 0$$

- For $W = const.$

$$\frac{d}{dt}\int_{D(t)} dx = \int_{\partial D(t)} \vec{\mathbf{w}_D} \cdot \vec{n} dS$$

$$\int_{t_n}^{t_{n+1}} \int_{D(t)} dx dt = \int_{t_n}^{t_{n+1}} \int_{\partial D(t)} \vec{\mathbf{w}_D} \cdot \vec{n} dS dt$$

Geometrical Conservation Law

For FVs:

- Use equation $W_t + \nabla \cdot F(W) = 0$

$$\frac{d}{dt}\int_{D(t)} W dx + \int_{\partial D(t)} \left(\vec{F}(W) - \vec{\mathbf{w}_D} W\right) \cdot \vec{n} dS = 0$$

- For $W = const.$

$$\frac{d}{dt}\int_{D(t)} dx = \int_{\partial D(t)} \vec{\mathbf{w}_D} \cdot \vec{n} dS$$

$$\int_{t_n}^{t_{n+1}} \int_{D(t)} dx dt = \int_{t_n}^{t_{n+1}} \int_{\partial D(t)} \vec{\mathbf{w}_D} \cdot \vec{n} dS dt$$

- For Finite Elements:
  ALE non-conservative scheme - satisfies GCL for approximation of $\mathbf{w}_D$

Weak Formulation

- The ALE derivative is approximated

$$\frac{D^{\mathcal{A}}\mathbf{v}}{Dt} \approx \frac{3\mathbf{v}_{n+1} - 4\tilde{\mathbf{v}}_n + \tilde{\mathbf{v}}_{n-1}}{2\Delta t}$$

- Weak formulation: find $\mathbf{v}_{n+1} = \mathbf{v}$, p

Weak formulation

$$\left(\frac{3\mathbf{v}}{2\Delta t}, \mathbf{z}\right) + \left([(\mathbf{v} - \mathbf{w}_g) \cdot \nabla]\,\mathbf{v}, \mathbf{z}\right) + \nu(\nabla\mathbf{v}, \nabla\mathbf{z})$$
$$-(p, \nabla^T \cdot \mathbf{z}) + (\nabla \cdot \mathbf{v}, q) = \left(\frac{4\tilde{\mathbf{v}}_n - \tilde{\mathbf{v}}_{n-1}}{2\Delta t}, \mathbf{z}\right)$$

Galerkin formulation

- anisotropic triangular meshes (refined in boundary layers)
- P2/P1 or P1/P1 elements

FEM gives unstable results?

- Galerkin method is unstable $\rightarrow$ several sources of instabilities
- Babuška-Brezzi inf-sup condition

$$c > 0, \qquad , \forall q_h, \qquad \sup_{\mathbf{v}_h} \frac{(q_h, \nabla \cdot \mathbf{v}_h)}{\|\mathbf{v}_h\|_{1,2,\Omega}} \geq c\|q_h\|_{0,2,\Omega}$$

- very high Reynolds numbers $\rightarrow$ convection dominated flows $Re_K^{loc} = \frac{h\|\mathbf{v}\|_K}{\nu} > 1$

Spatial discretization

- Stabilization $\psi = (\mathbf{w} \cdot \nabla)\mathbf{z} + \nabla q$

$$\mathcal{L}(W; U, V) = \sum_K \delta_K \left(\frac{3}{2\Delta t}\mathbf{v} - \nu \triangle \mathbf{v} + (\mathbf{w} \cdot \nabla)\mathbf{v} + \nabla p, \psi \right)_K,$$

$$\mathcal{F}(W; V) = \sum_K \delta_K \left(\frac{4\mathbf{v}^n - \mathbf{v}^{n-1}}{2\Delta t}, \psi \right)_K,$$

Stabilized problem

Galerkin terms, GLS stabilization, grad-div stabilization

$$a(U; U, V) + \mathcal{L}(U; U, V) + \sum_{K \in \tau_h} \tau_K \left(\nabla \cdot \mathbf{v}, \nabla \cdot \mathbf{z} \right)_K = f(V) + \mathcal{F}(U; V).$$

Aeroelastic computations (2DOF)

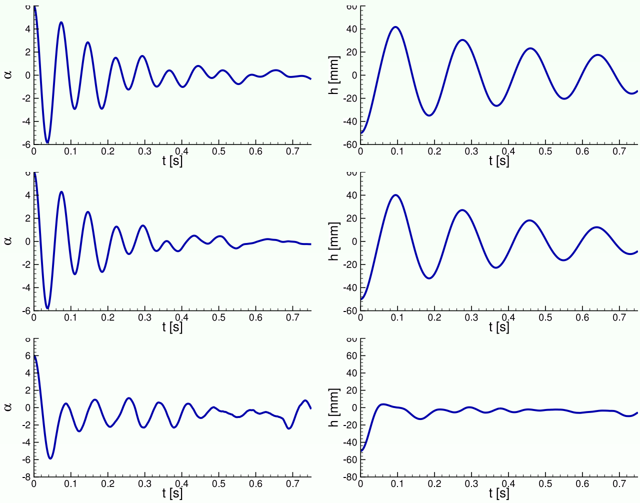


Figure: $U_\infty = 6, 8, 30$ m/s

Aeroelastic computations (2DOF)

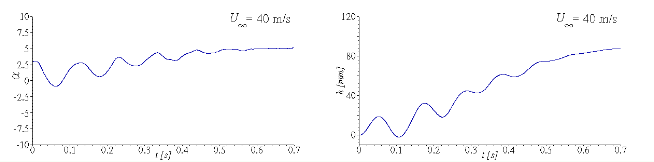


Figure: $U_\infty = 40$ m/s

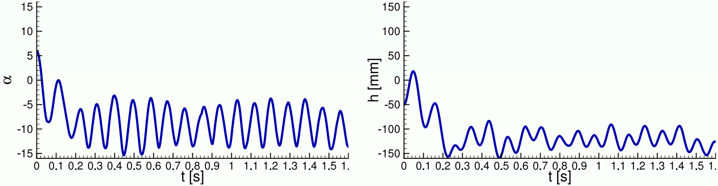


Figure: $U_\infty = 42$ m/s

Aeroelastic computations

- flexibly supported airfoil NACA $63_2 - 415$
- U=42 m/s, super-critical speed, **AVI format**

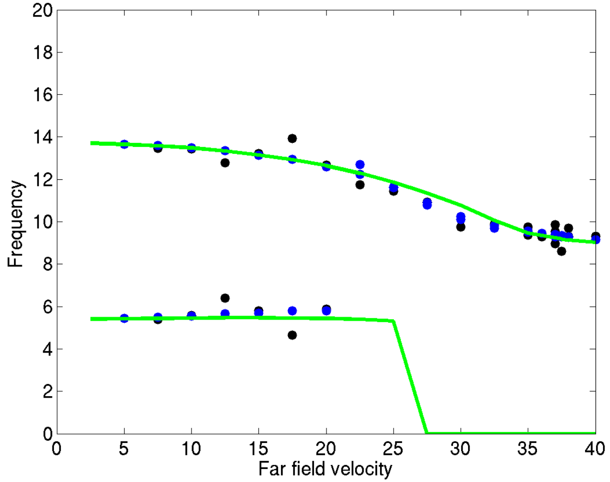
Aeroelastic model 2DOF

- flexibly supported airfoil NACA 0012
- RANS + Spallart-Almaras turbulence model
- Theodorsen theory critical speed $U_\infty = 37.7 m/s$
- frequencies and damping comparison

Aeroelastic model

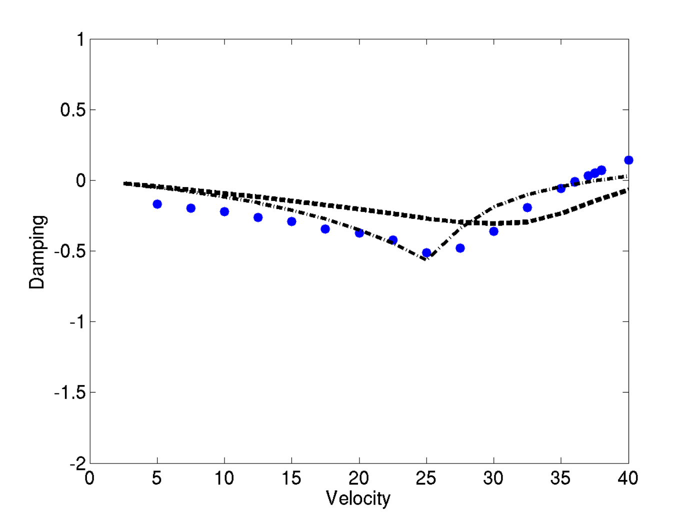
- U=37 m/s
- velocity isolines, **AVI format**

Comparison with Theodorsen (2DOF)



Frequencies comparison

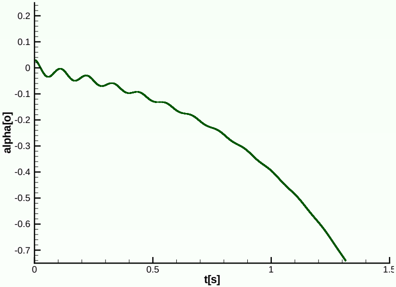
Comparison with Theodorsen (2DOF)

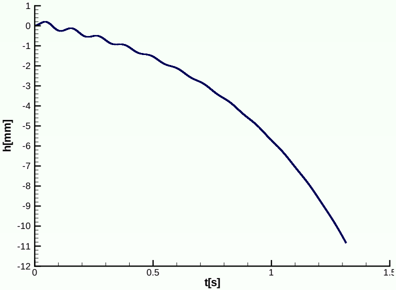


Damping

Nonlinear effects - effect of initial conditions

$h(0) = 0$ mm, $\alpha(0) = 0.03^\circ$

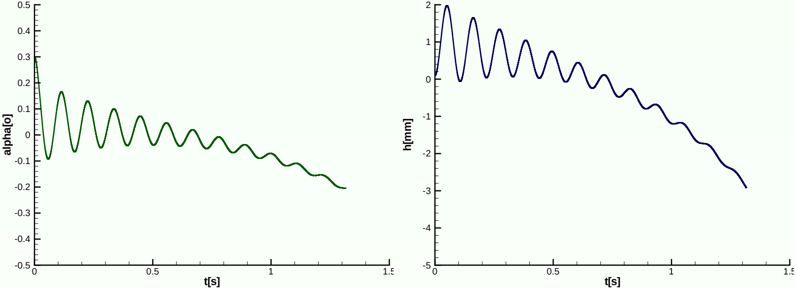




Post flutter simulations $U_\infty = 40m/s$, divergence type of instability, effect of initial conditions.

Nonlinear effects

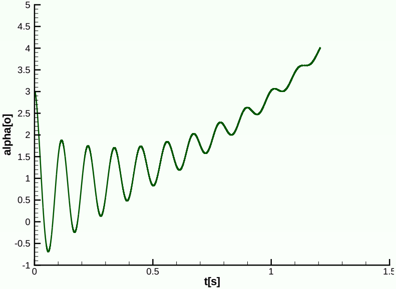
$h(0) = 0$ mm, $\alpha(0) = 0.3^\circ$

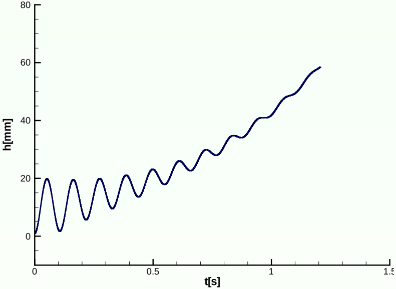


Post flutter simulations $U_\infty = 40m/s$, divergence type of instability, effect of initial conditions.

Nonlinear effects

$h(0) = 0$ mm, $\alpha(0) = 3^\circ$

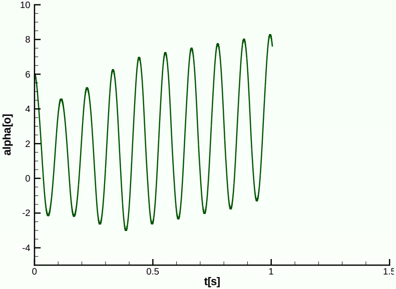


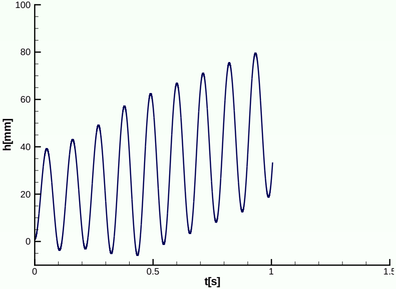


Post flutter simulations $U_\infty = 40m/s$, divergence type of instability, effect of initial conditions.

Nonlinear effects

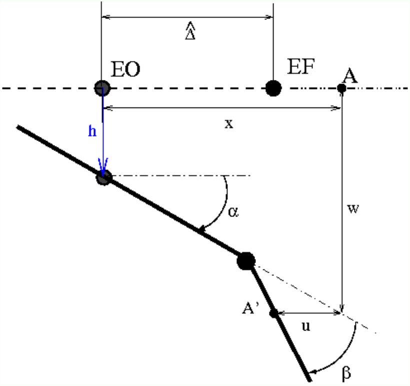
$h(0) = 0$ mm, $\alpha(0) = 6^\circ$





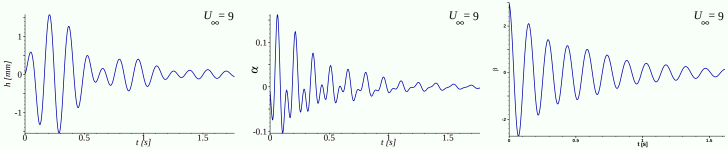
Post flutter simulations $U_\infty = 40m/s$, divergence type of instability, effect of initial conditions.

Aeroelastic computations 3DOF



- comparison of results with Theodorsen theory and NASTRAN Double-lattice model.

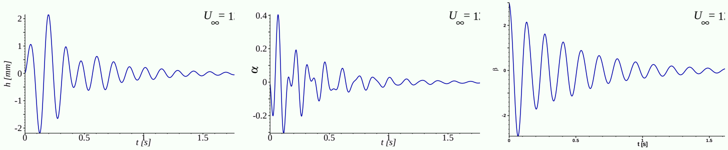
Aeroelastic computations 3dof



Solution of the coupled aeroelastic model (h,α,β), $U = 9m/s$

Back

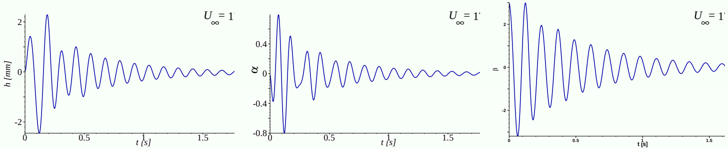
Aeroelastic computations 3dof



Solution of the coupled aeroelastic model (h, α, β), $U = 13 m/s$

Back

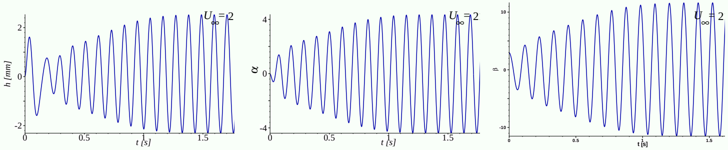
Aeroelastic computations 3dof



Solution of the coupled aeroelastic model (h,α,β), $U = 17m/s$

Back

Aeroelastic computations 3dof



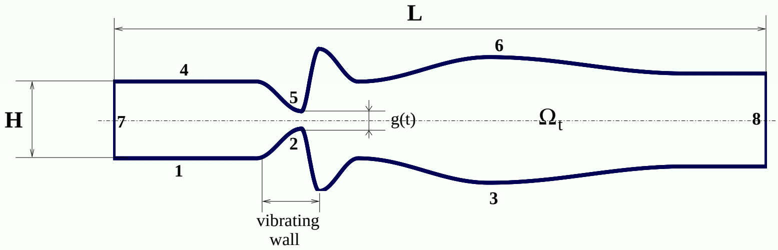
Solution of the coupled aeroelastic model (h,α,β), $U = 21m/s$

Back

Aeroelastic computations 3dof

Geometry

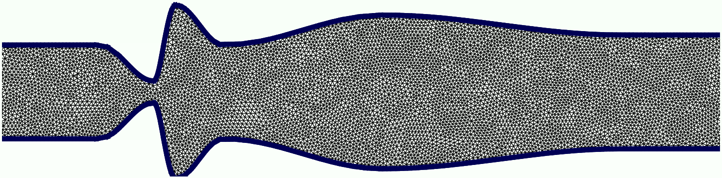
A simplified model of glottal region of human vocal tract

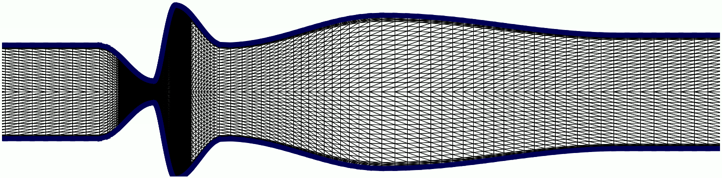


Time dependent computational domain (2d)

Stationary solution

- Inlet: given velocity
- gap - fixed, Re = 10-100
- grid isotropic (12219 vertices, 23709 elements, 8×10^4 unknowns)
- grid symmetric (8241 vertices, 16000 elements, 6×10^4 unknowns)

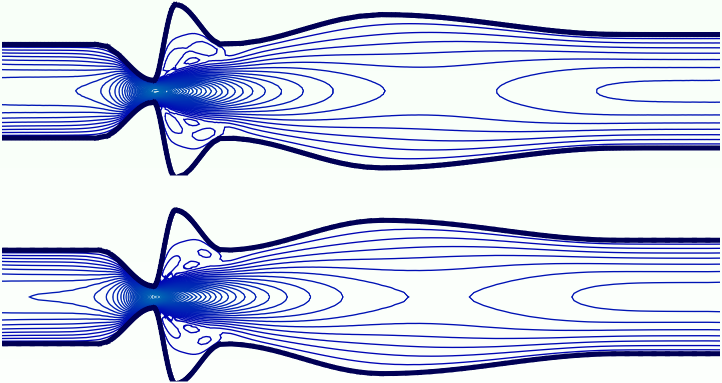




Stationary solution

Re = 20

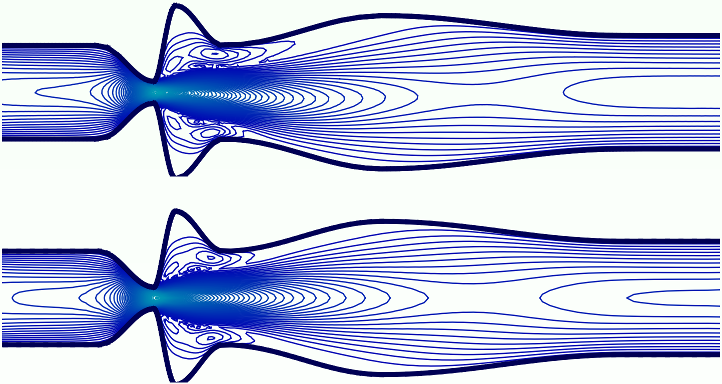
- grid isotropic (12219 vertices, 23709 elements, 8×10^4 unknowns)
- grid symmetric (8241 vertices, 16000 elements, 6×10^4 unknowns)



Stationary solution

Re = 30

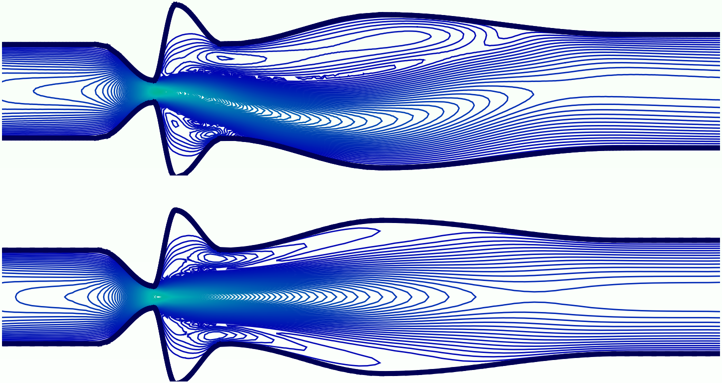
- grid isotropic (12219 vertices, 23709 elements, 8×10^4 unknowns)
- grid symmetric (8241 vertices, 16000 elements, 6×10^4 unknowns)



Stationary solution

Re = 40

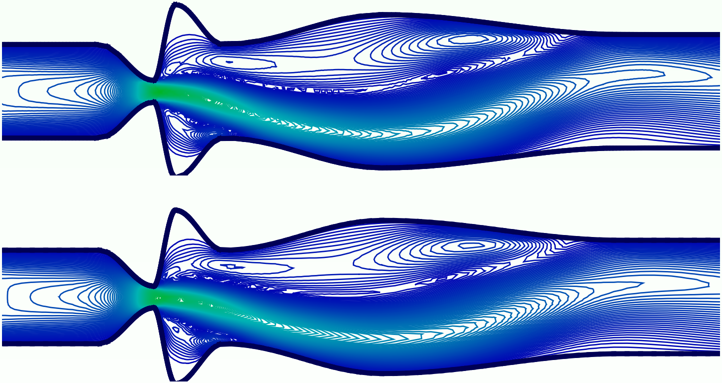
- grid isotropic (12219 vertices, 23709 elements, 8×10^4 unknowns)
- grid symmetric (8241 vertices, 16000 elements, 6×10^4 unknowns)



Stationary solution

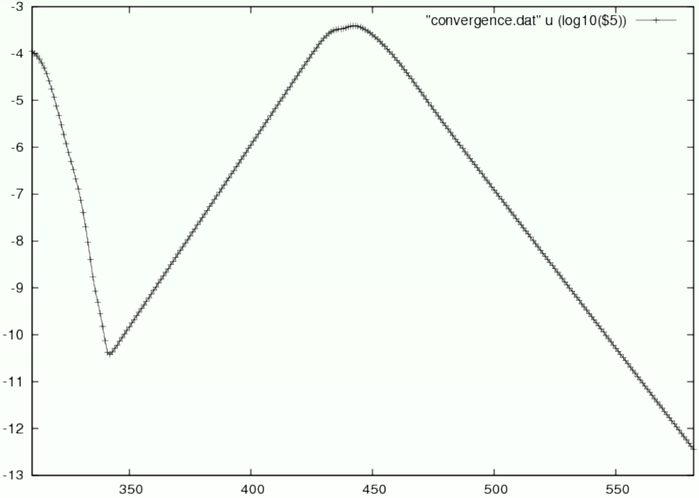
Re = 60

- grid isotropic (12219 vertices, 23709 elements, 8×10^4 unknowns)
- grid symmetric (8241 vertices, 16000 elements, 6×10^4 unknowns)



Stationary solution

Convergence for Re = 60

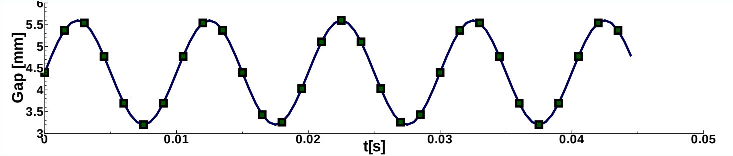


Numerical results

- Stationary solution (low Reynolds)
- **Non-stationary solutions** (multigrid method)

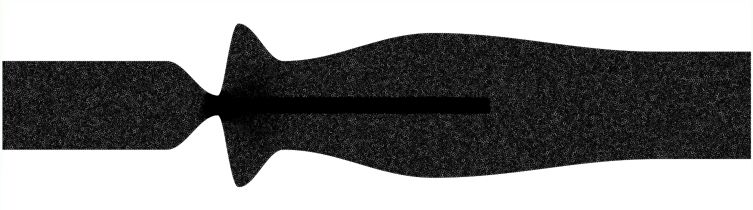
$$\Delta p = 400 \ Pa$$

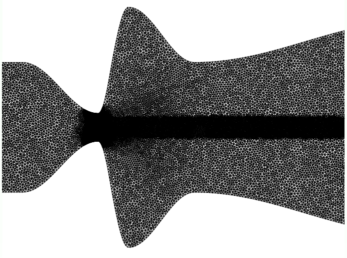
$$g(t) \in [3.2 \text{ mm}, 5.6 \text{ mm}], \ \text{frequency } f = 100 \ Hz.$$

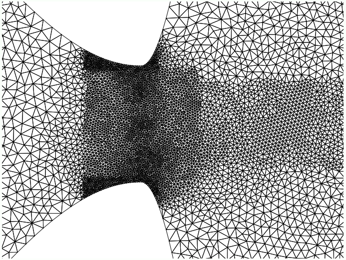


Non-stationary multigrid solution

- grid: 42576 vertices, 84078 elements, 4×10^5 unknowns

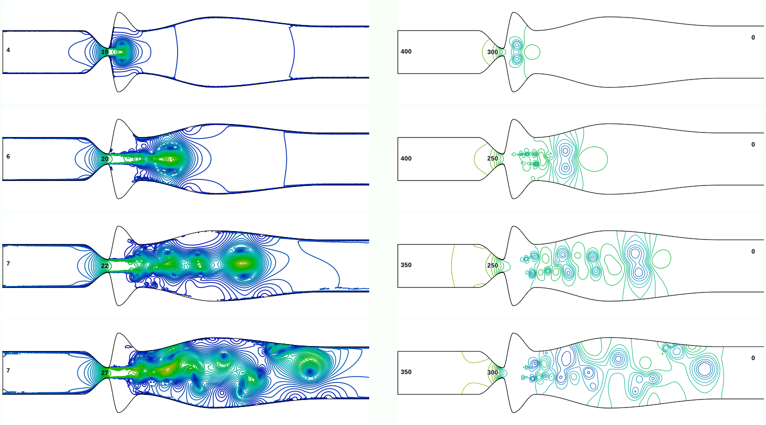






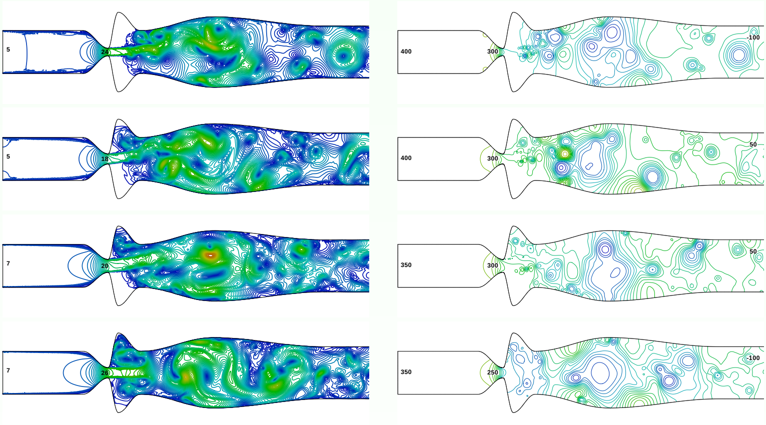
Non-stationary multigrid solution

- grid: 42576 vertices, 84078 elements, 4×10^5 unknowns



Non-stationary multigrid solution

- grid: 42576 vertices, 84078 elements, 4×10^5 unknowns
- Movie 1(vel) Movie 2(vel) Movie 3(p) Movie 3(vort)



Conclusion

- Byl popsáno několik typů problémů FSI, podrobně uveden
- Byl uveden popis matematického modelu, jednoduché odhady, popis numerického modelu a ukázky řešených problémů.

Další pokračování:

- modelování hlasivky včetně šíření zvukových vln
- 3D problém
- zahrnutí strukturálních nelinearit